Compromise, Don't Optimize: A Prior-Free Alternative to Bayesian Nash Equilibrium

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Abstract. Bayesian Nash equilibrium is the classic solution concept for games with incomplete information. Yet choices may be very suboptimal when the players' priors do not reflect true likelihoods, when players look in hindsight at events that occurred, or when players must justify their choices in front of others with different priors. We propose an alternative solution concept, best compromise equilibrium, that involves making choices that are approximately optimal for all priors. It is simple to calculate and, unlike dominant strategy and ex post Nash equilibrium, it always exists. We provide examples, including bilateral trade, public good provision, bargaining with incomplete information, and Cournot games with uncertain costs.

Preliminary draft

1. Introduction

A standard concept that extends Nash equilibrium to simultaneous games with incomplete information is Bayesian Nash equilibrium. Each player assigns a prior to the types of the other players and then chooses according to this prior a best response to the strategies of the others. In practice, players who face uncertainty rarely come up with a common prior and derive which choices are optimal under that prior. Even if they do, equilibrium strategies are often very complex and nonrobust in the prior and the common knowledge of the prior.

We propose a new equilibrium concept for games with incomplete information and call it *best compromise equilibrium*. Uncertainty means that each player is confronted with different possible situations. The classic approach would be to assign a probability to each of the different contingencies and to choose a best response decision as if these probabilities were objectively true. We prefer to consider players who make a

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choice that does well in each of the contingencies. As the same choice will not be optimal in all situations, these players will be compromising by making an epsilon-optimal choice. We assume that each player looks for the choice that makes the epsilon as small as possible, so never is too far from the optimal choice in any of the contingencies. We refer to this as a best compromise. A profile of strategies where each player chooses a best compromise against the play of the others is called a best compromise equilibrium.

Eliminating the need for a specific priors has several advantages. Solutions are often easier to obtain. Solutions are more parsimonious as they do not change with a prior. Solutions are more intuitive as they are simple and depend on concrete model primitives.

Our approach does not require the introduction of methods of non-expected utility maximization. The evaluation of the value of a strategy under a given prior is as under Bayesian Nash equilibrium. The term "compromise" reflects the fact that the equilibrium strategy will be considered close to optimal by players who have different priors. The adjective "best" is added as we look for the strategy that minimizes the loss from not using the optimal prior, where this minimum is taken across all priors.

Under standard compactness and continuity assumptions, our best compromise equilibrium always exists. This is in contrast to other equilibrium concepts without priors, such as dominant strategy equilibrium and ex post Nash equilibrium, which often do not exist, as in Cournot competition with uncertain costs (see Section 6). Even if they exist, the demanding condition of best responding for any type realization can limit efficiency, as in the bilateral trade game (see Section 3). However, similarly to ex post Nash equilibrium, our concept is an extension of Nash equilibrium to incomplete information games and coincides with Nash equilibrium when the information is complete.

We derive best compromise equilibria in a variety of situations including bilateral trade, public good provision, bargaining and Cournot competition. Let us illustrate our approach by showing how we solve the bilateral trade problem. Consider the classic setting where a seller of an indivisible good faces a single buyer. Each participant knows her own value of the good, but not that of their counterpart. Assume that trade is organized as in Chatterjee and Samuelson (1983) where each party makes a bid, and trade occurs at a price equal to the midpoint if their bids are compatible. How should the traders choose their bids? Using the Bayesian Nash approach, it is not enough to contemplate on how valuable the good is to the other side, one also has to think about what they believe your own value is. Bayesian Nash equilibrium can solve the game if the priors are already given. But we have no theory on how to derive priors. So there is no rule for how to bid, one cannot say that Bayesian Nash equilibrium solves the problem. Now consider our approach where each side understands this and tries to accommodate all possibilities. Each side knows the possible range of values of the other side, our equilibrium concept then reveals what each type should bid in order not lose too much in every contingency. The strategy is simple. Each trader bids a constant plus 2/3 of her own value, where the buyer's and seller's constants differ.[1] This typically does not constitute a best response to the true behavior of the opponent. But it guarantees that one loses at most 1/4 of her maximal payoff. Loss of at most 1/4 of the maximal payoff is a small price for not having to contemplate about the value of the other and for being protected against any misunderstandings.

Related Literature. Best compromise equilibrium can be considered a generalization of ex post Nash equilibrium. It can be thought of as an ε-ex post Nash equilibrium in which the smallest value of ε is chosen for each player. As a concept, ε-Nash equilibrium is usually seen as a play under the restriction that deviations are only undergone if payoff improvements are substantial. Our interpretation is different: ε does not measure the inertia that needs to be overcome, but instead it measures the compromise needed to accommodate all priors. The value of ε can also be interpreted as the cost of robustness. So, one can interpret best compromise equilibria as globally robust versions of Bayesian Nash equilibria where robustness (e.g., Huber, 1965) means to make choices that also perform well if the model is slightly misspecified, as in Stauber (2011). In this sense, the objectives used in our concept are close in spirit to those used in Bayesian Nash equilibrium. This differentiates us from other approaches to game playing under incomplete information and ambiguity that introduce new preferences for dealing with multiple priors, such as maximin utility preferences (Kajii and Morris, 1997) or so-called smooth ambiguous preferences (Klibanoff, Marinacci and Mukerji, 2005; Hanany, Klibanoff and Mukerji, 2018). In particular, in the maximin utility approach, it is as if one is focusing on a particular prior, ignoring all others as these lead to higher payoffs. In contrast, we take the opposite approach and deal with all priors simultaneously. Ambiguity preferences as considered in Hanany, Klibanoff and Mukerji (2018) can lead to new outcomes, even if ambiguity is not payoff relevant.

Another difference is that Hanany, Klibanoff and Mukerji (2018) also consider sequential decision making, something we left out of this paper to keep presentation simple. Our paper builds on simple mathematics that leads to clean and simple predictions in relevant games like bilateral trade (see Section 3). We leave the question of applying best compromise to extensive form games for future research.

We remain true to the idea of Nash equilibrium that precludes any strategic uncertainty. In other words, there is common knowledge of the strategy used by each type of each player. Each player would be certain about the play of the others had she known their private information. This contrasts our best compromise equilibrium with solution concepts that deal strategic uncertainty by securing a certain performance level against all possible actions of other players without justification behind these actions, such as maximin utility (Von Neumann and Morgenstern, 1953) and minimax regret

[1] This is different from the equilibrium in Linhart and Radner (1989) where each player's minimax regret is evaluated not over all possible types, but over all possible bids of the other side, even if these bids cannot occur in equilibrum.

(Savage, 1951; Linhart and Radner, 1989), as well as solution concepts that assume bounded rationality of players, such as quantal response equilibrium (McKelvey and Palfrey, 1995). These equilibrium concepts, unlike best compromise equilibrium, often do not coincide with Nash equilibrium in the complete information games, except, perhaps, in special subclasses of games, such as maximin utility applied to zero-sum games.

Our best compromise equilibrium provides an epsilon-best response against all possible priors. This is different from Stauber (2011) who tests robustness of Bayesian Nash equilibrium by perturbing each player's priors and from Renou and Schlag (2010) who assume that, with a small probability, each player chooses an arbitrary action.

2. Solution Concept

2.1. **Best Compromise Equilibrium.** Consider an n-player simultaneous move game. Each player $i \in \{1, ..., n\}$ has a private type $t_i \in T_i$ and chooses an action $a_i \in A_i$. Each player i has a continuous payoff function $u_i : T \times A \to \mathbb{R}$, where $T = \times_{i=1}^n T_i$ and $A = \times_{i=1}^n A_i$. For each i, the set of actions A_i is a compact convex metric space and the set of types T_i is a compact metric space.

Denote each player i's strategy by $s_i : T_i \to A_i$. Fix a player i and a strategy profile s_{-i} of the other players, where $s_{-i}(t_{-i}) = (s_j(t_j))_{j \neq i}$. The difference

$$\sup_{x \in A_i} u_i(t_i, t_{-i}, x, s_{-i}(t_{-i})) - u_i(t_i, t_{-i}, a_i, s_{-i}(t_{-i})),$$

is called player i's *complaint* about action a_i under a type profile $(t_i, t_{-i}) \in T$. It describes how much better payoff player i could have obtained if she chose the optimal action instead of action a_i under the realized type profile.

Player i's *maximum complaint* about action a_i is

$$c(t_i, a_i, s_{-i}) = \sup_{t_{-i} \in T_{-i}} \Big[\sup_{x \in A_i} u_i(t_i, t_{-i}, x, s_{-i}(t_{-i})) - u_i(t_i, t_{-i}, a_i, s_{-i}(t_{-i})) \Big]$$

A strategy s_i is called a *best compromise* to s_{-i} if it minimizes the maximum complaint:

$$s_i(t_i) \in BC_i(t_i) = \arg\min_{a_i \in A_i} c(t_i, a_i, s_{-i}) \quad \text{for all } t_i \in T_i.$$

A strategy profile $(s_1, ..., s_n)$ is called a *best compromise equilibrium* if s_i is a best compromise to s_{-i} for each $i \in \{1, ..., n\}$.

In this model we consider only pure strategies. This, however, includes finite games with mixed strategies as a special case. For each a player i's finite set of pure strategies, we denote by A_i the set of i's mixed strategies, and u_i is the linear extension of player i's payoff function to the mixed strategy space. In games with a continuum of strategies, A_i can represent the set of player i's pure strategies when mixed strategies are not allowed, or when the payoff function of each player is concave in her strategy (so that mixed strategies are inferior to pure strategies and, thus, can be ignored).

We now show the existence of a best compromise equilibrium.

Proposition 1. *A best compromise equilibrium exists.*

Proof. Because u_i is continuous and defined on a product of compact metric spaces, the maximum complaint function $c(t_i, a_i, s_{-i})$ is continuous. Because action space A_i is convex, the best compromise correspondence $BC_i(s_{-i}) = \arg\min_{a_i \in A_i} c(t_i, a_i, s_{-i})$ is nonempty, convex, and upper hemicontinuous. Thus, by Kakutani-Fan-Glicksberg fixed point theorem (Fan, 1952; Glicksberg, 1952), there exists a fixed point of the mapping $(BC_i(s_{-i}))_{i\in\{1,\ldots,n\}}$. □

2.2. **Properties of Best Compromise Equilibrium.**

Relation to Nash Equilibrium. Best compromise equilibrium is an extension of Nash equilibrium to games with incomplete information. Observe that in a game with complete information, where $T_i = \{t_i\}$ is a singleton for each i, action $s_i(t_i)$ minimizes the maximum complaint $c(t_i, s_i(t_i), s_{-i})$ if and only if this action is a best response to s_{-i}. Thus, a strategy profile s is a best compromise equilibrium if and only if it is a Nash equilibrium.

Moreover, best compromise equilibrium can be considered as a generalization of ex post Nash equilibrium. A strategy profile $s = (s_1, ..., s_n)$ is an ε-*ex post Nash equilibrium* for $\varepsilon \geq 0$ if, for all $i \in \{1, ..., n\}$ and all $(t_i, t_{-i}) \in T$,

$$u_i(t_i, t_{-i}, s_i(t_i), s_{-i}(t_{-i})) \geq \max_{x\in A_i} u_i(t_i, t_{-i}, x, s_{-i}(t_{-i})) - \varepsilon,$$

In words, in an ε-ex post Nash equilibrium, each player i's ex post payoff is always within ε of the best-response ex post payoff, where "ex post" means after all the uncertainty has been resolved, so each type t_i becomes known to everyone.

Let s be a best compromise equilibrium, and let $\varepsilon \geq 0$ be a number that exceeds all equilibrium complaints of all players,

$$\varepsilon \geq c(t_i, s_i(t_i), s_{-i}) \text{ for all } t_i \in T_i \text{ and all } i \in \{1, ..., n\}.$$

Clearly, a best compromise equilibrium s is an ε-ex post Nash equilibrium with the smallest ε that satisfies the above property.

Dominance. The concept of best compromise equilibrium respects dominance and iterated dominance, just as Nash equilibrium does in games with complete information, and Bayesian Nash equilibrium does in games with incomplete information.

We say that an action a'_i is *weakly dominated* by an action a''_i if

$$u_i(t_i, t_{-i}, a'_i, a_{-i}) \leq u_i(t_i, t_{-i}, a''_i, a_{-i}) \text{ for all } (t_i, t_{-i}) \in T_{-i} \text{ and all } a_{-i} \in A_{-i},$$

and the inequality is strict for some $(t_i, t_{-i}) \in T$ and some $a_{-i} \in A_{-i}$. We say that a'_i is *strictly dominated* by a''_i if the above inequality is strict for all $(t_i, t_{-i}) \in T$ all $a_{-i} \in A_{-i}$. Iterated dominance is defined in a standard way: after having excluded actions dominated in previous rounds, one checks the dominance condition w.r.t. the remaining actions of each player.

Observe that if an action a_i' is strictly dominated, then it cannot minimize the maximum complaint $c(t_i, a_i, s_{-i})$, and thus it cannot be a part of any best compromise equilibrium. This argument can be iterated, so any iterated strictly dominated action cannot be a part of any best compromise equilibrium.

Also, similarly to weak dominance in complete information games, iterated deletion of weakly dominated actions cannot eliminate all best compromise equilibria of the original game. Indeed, let a'' weakly dominate a'. If a' minimizes the maximum complaint $c(t_i, a_i, s_{-i})$, so does a'', and thus elimination of a' does not change the value of the minimax complaint.

Best Compromise against Bayesian Priors. A best compromise equilibrium obtains a minimax complaint against all possible realized types of the other players. We now point out that it also obtains the same minimax complaint against all possible Bayesian priors about these types.

For each player i, let $\Delta(T_{-i})$ be the set of Borel probabilities on $T_{-i} = \times_{j \neq i} T_j$, so $\mu_{-i} \in \Delta(T_{-i})$ is a prior about types of all players except i. For a given prior μ_{-i}, we write $E_{\mu_{-i}}[\cdot]$ for the expectation with respect to this prior.

Let $U_i(t_i, \mu_{-i}, a_i, s_{-i})$ be the expected payoff of player i with type t_i and action a_i under a given strategy profile s_{-i} of the other players and a given prior μ_{-i}, so

$$U(t_i, \mu_{-i}, a_i, s_{-i}) = \mathbb{E}_{\mu_{-i}}\big[u_i(t_i, t_{-i}, a_i, s_{-i}(t_{-i}))\big].$$

The payoff difference

$$\sup_{x \in A_i} U(t_i, \mu_{-i}, x, s_{-i})] - U(t_i, \mu_{-i}, a_i, s_{-i})$$

is called player i's *Bayesian complaint* about action a_i under a given strategy profile s_{-i} and a given prior μ_{-i}. It describes how much better expected payoff player i could have obtained if she chose the optimal action instead of action a_i under the prior μ_{-i}.

Player i's *maximum Bayesian complaint* about action a_i is

$$C(t_i, a_i, s_{-i}) = \sup_{\mu_{-i} \in \Delta(T_{-i})} \big[\sup_{x \in A_i} U(t_i, \mu_{-i}, x, s_{-i})] - U(t_i, \mu_{-i}, a_i, s_{-i})\big].$$

The next claim shows that, the maximum Bayesian complaint about strategy s_i is the same as the maximum complaint as defined in the previous section.

Claim 1. *For each i and each $t_i \in T_i$,*

$$C(t_i, a_i, s_{-i}) = c(t_i, a_i, s_{-i}).$$

Proof. Trivially, $C(t_i, a_i, s_{-i}) \geq c(t_i, a_i, s_{-i})$, because $\Delta(T_{-i})$ includes all degenerate priors in T_{-i}. The inequality $C(t_i, a_i, s_{-i}) \leq c(t_i, a_i, s_{-i})$ follows from

$$\begin{aligned} C(t_i, a_i, s_{-i}) &= \sup_{\mu_{-i} \in \Delta(T_{-i})} \Big[\sup_{x \in A_i} \mathbb{E}_{\mu_{-i}} \big[u_i(t_i, t_{-i}, x, s_{-i}(t_{-i})) \big] - \mathbb{E}_{\mu_{-i}} \big[u_i(t_i, t_{-i}, a_i, s_{-i}(t_{-i})) \big] \Big] \\ &= \sup_{x \in A_i} \sup_{\mu_{-i} \in \Delta(T_{-i})} \mathbb{E}_{\mu_{-i}} \big[u_i(t_i, t_{-i}, x, s_{-i}(t_{-i})) - u_i(t_i, t_{-i}, a_i, s_{-i}(t_{-i})) \big] \\ &\leq \sup_{x \in A_i} \sup_{t_{-i} \in T_{-i}} \big[u_i(t_i, t_{-i}, x, s_{-i}(t_{-i})) - u_i(t_i, t_{-i}, a_i, s_{-i}(t_{-i})) \big] = c(t_i, a_i, s_{-i}). \end{aligned}$$

□

2.3. **Informational vs Strategic Uncertainty.** A player's uncertainty is decomposed into two types: *informational* and *strategic*. Informational uncertainty is about not knowing private information of others, whereas strategic uncertainty is about not knowing what actions others will choose after all informational uncertainty has been resolved.

The problem of strategic uncertainty arises when a player treats the game as a decision problem against nature, so the behavior of the others is unmotivated and unpredictable. This problem is addressed by the solution concept of *dominant strategy*. In games where dominant strategies do not exist, the concept of *minmax regret* can be used instead. A minmax regret action can be interpreted as an ε-dominant action, where ε is the value of the minmax regret. In our notations, an action $a_i^*(t_i)$ is a minmax regret action if

$$a_i^*(t_i) \in \arg\min_{a_i \in A_i} \max_{\substack{t_{-i} \in T_{-i}, \\ a_{-i} \in A_{-i}}} \left[\max_{x \in A_i} u_i(t_i, t_{-i}, x, s_{-i}(t_{-i})) - u_i(t_i, t_{-i}, a_i, s_{-i}(t_{-i})) \right].$$

In contrast to our notion of maximum complaint, to find the maximum regret one needs to maximize the complaint over not only all types but also all actions of the others, whether or not these actions can emerge in equilibrium. Importantly, in games of complete information, minmax regret action profiles is often different from a Nash equilibrium.

In this paper, we remain true to the idea of Nash equilibrium that precludes any strategic uncertainty. So, we assume strategic certainty and focus on informational uncertainty that comes from private information of other players. If a player knew what information others have, she would have known their actions. The problem of information uncertainty has been addressed by the solution concepts of dominant strategy equilibrium and ex post Nash equilibrium. As we noted above, best compromise equilibrium is a natural generalization of these concepts, as every best compromise equilibrium can be interpreted as an ε-ex post Nash equilibrium. If an ex post Nash equilibrium or a dominant strategy equilibrium exist, these equilibria are best compromise equilibria with zero complaints.

2.4. **Efficiency Measures.** For comparison of best compromise equilibria in applications, we would like to introduce two measures: the maximal individual complaint and the maximal social complaint.

Let $s^* = (s_1^*, ..., s_n^*)$ be a best compromise equilibrium. The individual complaint of player i measures how close i's payoff is guaranteed to be to the best-response payoff under any prior. To obtain a scale-free measure, we normalize the payoffs as follows. Let $\underline{u}_i(t_i)$ be player i's *individually rational* payoff, so

$$\underline{u}_i(t_i) = \sup_{a_i \in A_i} \left(\inf_{a_{-i} \in A_{-i}, t_{-i} \in T_{-i}} u_i(t_i, t_{-i}, a_i, a_{-i}) \right).$$

This is the least payoff that player i with type t_i can guarantee no matter what. Also, let $\overline{u}_i(t_i)$ be the maximal possible payoff of player i with type t_i, so

$$\overline{u}_i(t_i) = \sup_{a_i \in A_i} \left(\sup_{a_{-i} \in A_{-i}, t_{-i} \in T_{-i}} u_i(t_i, t_{-i}, a_i, a_{-i}) \right).$$

The *maximal normalized individual complaint* of player i with type t_i is

$$\varepsilon_i(t_i, s^*) = \frac{c_i(t_i, s_i^*(t_i), s_{-i}^*)}{\overline{u}_i(t_i) - \underline{u}_i(t_i)}.$$

The maximal social complaint measures the maximum efficiency loss as compared to the potential maximum of the sum of the payoffs of all players. Let $\bar{S}$ be the difference between the maximal possible aggregate payoff of all players and the sum of their individually rational payoffs, referred to as the *maximal surplus*,

$$\bar{S} = \sup_{t \in T} \left(\sup_{a \in A} \sum_{i=1}^{n} u_i(t, a) - \sum_{i=1}^{n} \underline{u}_i(t_i) \right).$$

The *maximal social complaint* is

$$\bar{C}(s^*) = \sup_{t \in T} \frac{\sup_{a \in A} \sum_{i=1}^{n} u_i(t, a) - \sum_{i=1}^{n} u_i(t, s^*(t))}{\bar{S}}.$$

3. Bilateral Trade

Consider a model of bilateral trade (Chatterjee and Samuelson, 1983). A seller has an indivisible good to sell to a potential buyer. The good has value v_s for the seller and v_b for the buyer. Each party is privately informed about own value, and it is commonly known that both values are in an interval $[0, \bar{v}]$.

The bargaining procedure is as follows. The seller and the buyer simultaneously submit bids, s and b, respectively. If $s \leq b$, then the good is sold at the price $p = (s + b)/2$, so the seller and the buyer obtain the payoffs of $p - v_s$ and $v_b - p$, respectively. If $s > b$, then the good is not sold, and both parties obtain zero payoffs.

3.1. **Bayesian Nash equilibrium.** Let v_s and v_b be distributed on $[0, \bar{v}]$ according to distributions F_s and F_b that admit densities f_s and f_b, respectively.

Chatterjee and Samuelson (1983) characterize Bayesian Nash equilibria under the following regularity condition:

(A_1) Strategies $s(v_s)$ and $b(v_b)$ are strictly increasing and differentiable almost everywhere.

Claim 2 (Chatterjee and Samuelson 1983, Theorem 2)**.** *A pair of strategies* (s, b) *that satisfies* (A_1) *is a Bayesian Nash equilibrium only if there exist bounds* $0 \le \underline{v}_s \le \overline{v}_s \le \bar{v}$ *and* $0 \le \underline{v}_b \le \overline{v}_b \le \bar{v}$ *such that for all* $v_s \in [\underline{v}_s, \overline{v}_s]$ *and all* $v_b \in [\underline{v}_b, \overline{v}_b]$

$$\frac{1}{2}F_b(v_s)s'(v_s) + f_b(v_s)s(v_s) = b^{-1}(s(v_s))f_b(v_s),$$

$$\frac{1}{2}(1 - F_s(v_b))b'(v_b) + f_s(v_b)b(v_b) = -s^{-1}(b(v_b))f_s(v_b),$$

and there is no trade when $v_s > \overline{v}_s$ *irrespective of* v_b*, and there is no trade when* $v_b < \underline{v}_b$ *irrespective of* v_s*.*

For general distributions, this system of differential equations does not have a closed form solution. Chatterjee and Samuelson (1983) solve this problem for uniform distributions, in particular, in the symmetric case of $F_b(x) = F_s(x) = x/\bar{v}$. With the appropriately chosen initial conditions, the solution is

$$s(v_s) = \max\left\{v_s, \frac{\bar{v}}{4} + \frac{2v_s}{3}\right\} \quad \text{and} \quad b(v_b) = \min\left\{v_b, \frac{\bar{v}}{12} + \frac{2v_b}{3}\right\}. \tag{1}$$

Note that $s(v_s) \ge \bar{v}/4$. So, the trade cannot occur if $v_b < \underline{v}_b = \bar{v}/4$. For the values of $v_b < \bar{v}/4$, there are multiple ways how to define equilibrium strategy $b(v_b)$ that induces no trade, so $b(v_b) < \bar{v}/4$. We choose $b(v_b) = v_b$ in this case. Similarly, for the values of $v_s > \overline{v}_s = 3\bar{v}/4$, there are multiple ways how to define equilibrium strategy $s(v_s)$ that induces no trade. We choose $s(v_s) = v_s$ in this case.

3.2. **Best compromise equilibrium.** We now find a best compromise equilibrium under the same regularity condition (A_1) imposed by Chatterjee and Samuelson (1983).

Claim 3. *A pair of strategies* (s^*, b^*) *that satisfies* (A_1) *is a best compromise equilibrium if and only if, for all* $v_s, v_b \in [0, \bar{v}]$*,*

$$s(v_s) = \max\left\{v_s, \frac{\bar{v}}{4} + \frac{2v_s}{3}\right\} \quad \textit{and} \quad b(v_b) = \min\left\{v_b, \frac{\bar{v}}{12} + \frac{2v_b}{3}\right\}. \tag{2}$$

The maximal normalized individual complaints in this equilibrium are given by

$$\varepsilon_s(v_s) = \max\left\{\frac{1}{4} - \frac{v_s}{12(\bar{v} - v_s)}, 0\right\} \quad \textit{and} \quad \varepsilon_b(v_b) = \max\left\{\frac{1}{4} - \frac{\bar{v} - v_b}{12v_b}, 0\right\}. \tag{3}$$

So, the best compromise equilibrium coincides with the Bayesian Nash equilibrium under the symmetric uniform prior. The maximal individual complaints in (3) show that, as compared to the best-response payoffs, each trader never loses more than 1/4 of her maximal surplus.

One could use the "principle of insufficient reasoning" to justify our choice of a solution concept. When you know nothing about information of other players, do not best respond to hypothetical unjustified priors, but instead find a best compromise. Interestingly, the "principle of insufficient reasoning" has often been used to justify uniform priors, which incidentally leads to the same equilibrium behavior as best compromise in the case of bilateral trade.

Proof. Denote by $s^*(v_s)$ and $b^*(v_b)$ the strategies of the seller and the buyer in a best compromise equilibrium. Also, denote

$$\underline{s}^* = \inf_{v_s \in [0,\bar{v}]} s^*(v_s) \quad \text{and} \quad \overline{b}^* = \sup_{v_b \in [0,\bar{v}]} b^*(v_b).$$

Suppose that the seller's value is v_s. Let us evaluate the seller's maximum complaint for bidding s. If $v_s < \overline{b}^*$, so there is a potential gain from trade when the buyer's bid $b^*(v_b)$ is high enough, then there are two outcomes that the seller worries about. First, it is $s < b^*(v_b)$, so the seller could have made a greater bid, $s' = b^*(v_b)$, and sold the good at a higher price, thus obtaining a payoff increment of

$$\sup_{v_b} \left[\left(b^*(v_b) - v_s \right) - \left(\frac{s + b^*(v_b)}{2} - v_s \right) \right] = \frac{\underline{b}^* - s}{2}.$$

Second, it is $s > b^*(v_b) > v_s$, so the good is not sold, but the seller could have made a smaller bid, $s' = b^*(v_b)$, and sold the good, thus obtaining a payoff increment of

$$\sup_{v_b} \left[b^*(v_b) - v_s \right] = s - v_s.$$

The seller's maximum complaint is thus

$$c_s(v_s, s, b^*) = \max \left\{ \frac{\underline{b}^* - s}{2}, s - v_s \right\} \quad \text{if } v_s < \overline{b}^*.$$

The value of s that minimizes the above maximum complaint is

$$s^*(v_s) = \frac{\overline{b}^*}{3} + \frac{2v_s}{3}, \quad \text{if } v_s < \overline{b}^*.$$

Alternatively, if there is no gain from trade, so $v_s \geq \bar{b}^*$, then the seller trivially obtains zero complaint by choosing any bid that guarantees no trade, for example, $s^*(v_s) = v_s$.

Symmetrically, we obtain that the buyer's best compromise strategy satisfies

$$b^*(v_b) = \begin{cases} \frac{\underline{s}^*}{3} + \frac{2v_b}{3}, & \text{if } v_b > \underline{s}^*, \\ v_b, & \text{if } v_b \leq \underline{s}^*. \end{cases}$$

Since $v_s, v_b \in [0, \bar{v}]$, we have

$$\underline{s}^* = s^*(0) = \frac{\overline{b}^*}{3} \quad \text{and} \quad \overline{b}^* = b^*(\bar{v}) = \frac{2\bar{v} + \underline{s}^*}{3}.$$

Thus, $\underline{s}^* = \bar{v}/4$ and $\overline{b}^* = 3\bar{v}/4$. The best compromise equilibrium (s^*, b^*) given by

$$s^*(v_s) = \max\left\{v_s, \frac{\bar{v}}{4} + \frac{2v_s}{3}\right\} \quad \text{and} \quad b^*(v_b) = \min\left\{v_b, \frac{\bar{v}}{12} + \frac{2v_b}{3}\right\}. \tag{4}$$

To derive the maximal normalized individual complaints, observe that the minimal guaranteed payoff of each trader is zero. The maximum possible payoff of the seller is $\bar{v} - v_s$ (as if the buyer buys at the maximum price $\bar{v}$), and the maximal possible payoff of the buyer is v_s (as if the seller sells at zero price). Substituting s^* and b^* into the maximum complaints c_s and c_b, and then dividing by the maximal possible payoff for a given type yields (3). □

3.3. **Ex post Nash equilibrium.** We now consider a family of best compromise equilibria that do not satisfy condition (A_1). These are ex post Nash equilibria of this game.

Any ex post Nash equilibrium is characterized by an exogenous price $p^{**} \in \mathbb{R}$. The good is sold if and only if the parties obtain surplus by trading at that price, so $v_s \leq p^{**} \leq v_b$. In particular, when p^{**} is outside of $[0, \bar{v}]$, then this is a no-trade equilibrium.

Claim 4. *A pair of strategies (s^{**}, b^{**}) is an ex post Nash equilibrium if and only if there exists a price $p^{**} \in \mathbb{R}$ such that*

$$\begin{aligned} &s^{**}(v_s) = b^{**}(v_b) = p^{**}, \quad \text{if } v_s \leq p^{**} \leq v_b, \\ &s^{**}(v_s) > p^{**}, \quad \text{if } v_s > p^{**}, \\ &b^{**}(v_b) < p^{**}, \quad \text{if } v_b < p^{**}. \end{aligned} \tag{5}$$

Proof. (Easy. To be completed.) □

There are multiple ways how to define equilibrium strategies $b^{**}(v_b)$ for $v_b < p^{**}$ and $s^{**}(v_s)$ for $v_s > p^{**}$ that lead to no trade. As previously, we choose $b^{**}(v_b) = v_b$ and $s^{**}(v_s) = v_s$ in these cases, thus (9) becomes

$$s^{**}(v_s) = \max\{v_s, p^{**}\} \quad \text{and} \quad b^{**}(v_b) = \min\{v_b, p^{**}\}. \tag{6}$$

Let us compare the efficiency of these ex-post Nash equilibria and the best compromise equilibrium (4). We measure the performance of an equilibrium (s, b) by the maximal social complaint as defined in Section 2.4. The maximum social surplus is $\bar{v}$. For each pair of types (v_s, v_b), when the trade occurs, so $b(v_b) \geq s(v_s)$, the complaint is

$\max\{0, v_s - v_b\}/\bar{v}$. Symmetrically, when the trade does not occur, so $b(v_b) < s(v_s)$, the loss is $\max\{0, v_b - v_s\}/\bar{v}$. Thus,

$$\bar{C}(s,b) = \frac{1}{\bar{v}} \sup_{(v_s,v_b)\in[0,\bar{v}]^2} \begin{cases} \frac{1}{\bar{v}} \max\{0, v_s - v_b\}, & \text{if } b(v_b) \geq s(v_s), \\ \frac{1}{\bar{v}} \max\{0, v_b - v_s\}, & \text{if } b(v_b) < s(v_s). \end{cases}$$

First, let us evaluate the efficiency loss for the best compromise equilibrium (s^*, b^*) given by (4). In this equilibrium, the trade occurs if and only if $b^*(v_b) \geq s^*(v_s)$, which can be simplified as

$$v_b - v_s \geq \frac{\bar{v}}{4}.$$

Thus, the maximum efficiency loss is $\bar{C}(s^*, b^*) = 1/4$.

Consider now the maximum efficiency loss of an ex-post Nash equilibrium with price p^{**}. The maximum efficiency loss is attained when one party has a lot to gain from trade, whereas the other party just misses the price p^{**}, so $v_b = \bar{v}$ and $v_s = p^{**} + \varepsilon$ or $v_b = p^{**} - \varepsilon$ and $v_s = 0$, where $\varepsilon \to 0$. So, the maximum efficiency loss is

$$\bar{C}(s^{**}, b^{**}) = \frac{1}{\bar{v}} \max\{\bar{v} - p^{**}, p^{**}\} \geq \frac{1}{2}.$$

We thus obtain that the maximum efficiency loss of any ex-post Nash equilibrium is at least twice as large as that of the best compromise equilibrium given by (4).

4. Bargaining

Consider Nash demand game. Two agents split a pie of value 1. Each agent $i = 1, 2$ demands $x_i \in [0, \bar{v}]$. If $x_1 + x_2 \leq 1$, then the agents split the pie at the midpoint between their demands, so agent i obtains

$$\frac{1}{2} + \frac{x_i - x_j}{2}.$$

If $x_1 + x_2 > 1$, then each agent i obtains her outside option v_i. We assume that v_i is private information of i, and it is commonly known that $v_i \in [0, 1]$.

Let $s_i(v_i)$ denote a strategy of agent i. We make the same assumption as in Section 3:

(A_2) Strategies $s_1(v_1)$ and $s_2(v_2)$ are strictly increasing and differentiable almost everywhere.

We now find a best compromise equilibrium.

Claim 5. *A pair of strategies (s_1^*, s_2^*) that satisfies (A_2) is a best compromise equilibrium if and only if, for each $i = 1, 2$ and each $v_i \in [0, 1]$,*

$$s_i^*(v_i) = \max\left\{v_i, \frac{1}{4} + \frac{2v_i}{3}\right\}. \tag{7}$$

The maximal normalized individual complaints in this equilibrium are given for each $i = 1, 2$ *and each* $v_i \in [0, 1]$ *by*

$$\varepsilon_i(v_i) = \max\left\{\frac{1}{4} - \frac{v_i}{12(1 - v_i)}, 0\right\}. \tag{8}$$

Proof. To be completed. Easy – this problem is invariant to the bilateral trade, up to a change of variables. □

We now consider a best compromise equilibrium that does not satisfy condition (A_2). This is a symmetric ex post Nash equilibrium of this game.

Claim 6. *A pair of strategies* $(\bar{s}_1, \bar{s}_2)$ *is a symmetric ex post Nash equilibrium if and only if*

$$\bar{s}_i(v_i) = \begin{cases} 1/2, & \text{if } v_i \le 1/2, \\ 1, & \text{if } v_i > 1/2. \end{cases} \tag{9}$$

Proof. (Easy. To be completed.) □

Let us compare the efficiency of the ex-post Nash equilibrium and the best compromise equilibrium (7). As before, we measure the efficiency of an equilibrium (s_1, s_2) by the maximal social complaint as defined in Section 2.4. The maximal social surplus is 1. When the pie is split, so $s_1(v_1) + s_2(v_2) \le 1$, the loss is $\max\{0, v_1 + v_2 - 1\}$. Symmetrically, when the pie is not split so $s_1(v_1) + s_2(v_2) > 1$, the loss is $\max\{0, 1 - v_1 - v_2\}$. So,

$$\bar{C}(s_1, s_2) = \sup_{(v_1, v_2) \in [0,1]^2} \begin{cases} \max\{0, v_1 + v_2 - 1\}, & \text{if } s_1(v_1) + s_2(v_2) \le 1, \\ \max\{0, 1 - v_1 - v_2\}, & \text{if } s_1(v_1) + s_2(v_2) > 1. \end{cases}$$

First, let us evaluate the efficiency of the best compromise equilibrium given by (7). In this equilibrium, the pie is split occurs if and only if $s_1^*(v_1) + s_2^*(v_2) \le 1$, which can be simplified as

$$v_1 + v_2 \le \frac{3}{4}.$$

Thus, the maximal social complaint is $\bar{C}(s^*, b^*) = 1/4$.

Consider now the efficiency of the ex-post Nash equilibrium. The maximum efficiency loss is attained when one party has zero outside option, whereas the other party has the outside option $1/2 + \varepsilon$ with $\varepsilon \to 0$. So, the maximal social complaint is

$$\bar{C}(\bar{s}_1, \bar{s}_2) = 1/2.$$

We thus obtain that the maximum efficiency loss of any ex-post Nash equilibrium is twice as large as that of the best compromise equilibrium given by (7).

5. Public Good Provision

Consider a problem of collecting contributions to pay for a public good. The cost of public good provision is $c > 0$. There are n agents, each has a private value $v_i \in [0, \bar{v}]$ for the public good. Each agent i commits to a maximal contribution of $x_i \in [0, \bar{v}]$ should the public good be provided. Agents make their commitments simultaneously.

If the total contribution does not cover the cost, so $\sum_{i=1}^n x_i < c$, then the public good is not provided, and each agent i obtains zero payoff. Otherwise, if $\sum_{i=1}^n x_i \geq c$, then the public good is provided, and each agent i obtains the payoff of

$$v_i - t_i(x),$$

where $t_i(x)$ is the final transfer that depends on the profile of contributions $x = (x_1, ..., x_n)$. We assume that

(a) $t_i(x) \leq x_i$, so no agent pays more than her committed contribution;

(b) $\sum_{i=1}^n t_i(x) \geq c$, so whenever the public good is provided, it is fully funded by the agents;

(c) transfer rule $t = (t_1, ..., t_n)$ is symmetric, so agents are treated ex ante equally.

In addition, we assume that the cost of public good provision is relatively small,

$$c \leq \frac{n-1}{2}\bar{v}. \tag{10}$$

This assumption is to simplify the exposition; the complementary case can also be easily analysed.

Denote by $x_i^*[0, \bar{v}] \to [0, \bar{v}]$ a strategy of agent i, so $x^*(v_i)$ is a contribution of agent i whose private value is v_i. As in Section 3, we restrict attention to strategies that satisfy the following assumption.

(A_3) Strategies $x_i^*(v_i)$ are strictly increasing and differentiable almost everywhere, and symmetric across agents.

We consider a few simple transfer rules.

(i) *Pay as you bid.* Each agent pays as much as she commits to contribute whenever the good is provided, so

$$t_i(x) = x_i. \tag{11}$$

(ii) *Proportional rule.* Each agent i pays proportionally to her commitment x_i whenever the good is provided, so

$$t_i(x) = \frac{cx_i}{\sum_{j=1}^n x_j}. \tag{12}$$

(iii) *Additive rule.* Suppose that each agent i pays according to the following rule:

$$t_i(x) = \frac{c}{n} + x_i - \frac{1}{n}\sum_{j=1}^n x_j. \tag{13}$$

The assumptions (a), (b), and (c) are easily verified for these transfer rules.

Claim 7. *A profile of strategies* $(x_1^*, ..., x_n^*)$ *that satisfy* (A_3) *is a best compromise equilibrium if and only if for all* $i = 1, ..., n$ *and all* $v_i \in [0, \bar{v}]$,

(i) if $t_i(x)$ *is given by* (11), *then*

$$x_i^*(v_i) = \frac{v_i}{2} \quad \text{and} \quad \varepsilon_i(v_i) = \frac{1}{2};$$

(ii) if $t_i(x)$ *is given by* (12), *then*

$$x_i^*(v_i) = \frac{v_i}{2} - c + \frac{1}{2}\sqrt{v_i^2 + 4c^2} \quad \text{and} \quad \varepsilon_i(v_i) = \frac{1}{2} + \frac{c}{v_i} - \frac{1}{2}\sqrt{1 + 4\left(\frac{c}{v_i}\right)^2};$$

(iii) if $t_i(x)$ *is given by* (13), *then*

$$x_i^*(v_i) = \frac{n}{2n-1} v_i \quad \text{and} \quad \varepsilon_i(v_i) = \frac{1}{2} - \frac{1}{2(2n-1)}.$$

Proof. An agent who chooses x_i worries about two contingencies. First, it could be that the total contribution is just below c, so $\sum_j x_j = c - \varepsilon$, so the good is not provided, but had i contributed ε more it would have been provided. So, as $\varepsilon \to 0$, agent i's complaint is $\max\{v_i - x_i, 0\}$.

Second, it could be that all other agents contribute enough to cover c, so $\sum_{j \neq i} x_j \geq c$, so the agent could have contributed nothing and still got the good. In this case the complaint is the amount of contribution, $t_i(x)$. Under all three assumptions, this complaint is maximized when the other agents' contributions exactly equal to the cost, so $\sum_{j \neq i} x_j = c$. If $t_i(x)$ is given by (11), then the complaint is x_i. If $t_i(x)$ is given by (12), then the complaint is $cx_i/(c + x_i)$. If $t_i(x)$ is given by (13), then the complaint is $(n-1)x_i/n$.

As the first type of complaint is weakly decreasing and the second type of complaint is strictly increasing in x_i, we equalize the two and solve for $x_i^*(v_i)$ that minimizes the maximum complaint. For each of the assumptions about $t_i(x)$, the solution $x_i^*(v_i)$ is given by (i), (ii), and (iii) in Claim 7.

The maximum individual surplus is v_i, so the maximal individual complaint $\varepsilon_i(v_i)$ is simply $\max\{v_i - x_i^*(v_i), 0\}/v_i = 1 - x_i^*(v_i)/v_i$. Substituting the obtained solutions $x_i^*(v_i)$ into this expression, we obtain $\varepsilon_i(v_i)$ in (i), (ii), and (iii) in Claim 7.

It remains to verify that there exist values v_j for each $j \neq i$ such that $\sum_{j \neq i} x^*(v_j) \geq c$. If $t_i(x)$ is given by (11), then

$$\max \sum_{j \neq i} x^*(v_j) = \max \sum_{j \neq i} \frac{v_j}{2} = \frac{(n-1)\bar{v}}{2},$$

If $t_i(x)$ is given by (12), then

$$\max \sum_{j\neq i} x^*(v_j) = \max \sum_{j\neq i} \left(\frac{v_j}{2} - c + \frac{1}{2}\sqrt{v_j^2 + 4c^2} \right)$$
$$= (n-1)\left(\frac{\bar{v}}{2} - c + \frac{1}{2}\sqrt{\bar{v}^2 + 4c^2} \right) \geq \frac{(n-1)\bar{v}}{2}.$$

Finally, if $t_i(x)$ is given by (13), then

$$\max \sum_{j\neq i} x^*(v_j) = \max \sum_{j\neq i} \frac{nv_j}{2n-1} = (n-1)\frac{n\bar{v}}{2n-1} \geq \frac{(n-1)\bar{v}}{2}.$$

Since $c \leq \frac{(n-1)\bar{v}}{2}$ by assumption (10), we obtain that there exist values v_j for each $j \neq i$ such that $\sum_{j\neq i} x^*(v_j) \geq c$. □

We measure the efficiency of the best compromise equilibrium for each choice of t_i by the maximal social complaint as defined in Section 2.4. The maximum possible surplus is $n\bar{v}$, and the individually rational value of each agent is 0. When the public good is not provided, the efficiency loss is $\max\{0, \sum_i v_i - c\}/(n\bar{v})$. When the public good is provided, the efficiency loss is $\max\{0, c - \sum_i v_i\}/(n\bar{v})$. Note that in Claim 7, $x_i^*(v_i) \leq v_i$, so the latter term is always zero. The only source of inefficiency is that the public good is not provided when it is efficient to do so,

$$\bar{C}(x^*) = \sup_{(v_1,\ldots,v_n)\in[0,\bar{v}]^n} \begin{cases} \frac{1}{n\bar{v}} \max\{0, \sum_i v_i - c\}, & \text{if } \sum_i x_i^*(v_i) \geq c, \\ 0, & \text{if } \sum_i x_i^*(v_i) < c. \end{cases}$$

Claim 8.

(i) If $t_i(x)$ *is given by* (11), *then*

$$\bar{C}(x^*) = \frac{1}{2};$$

(ii) if $t_i(x)$ *is given by* (12), *then*

$$\bar{C}(x^*) = \frac{n}{2n+1}$$

(iii) if $t_i(x)$ *is given by* (13), *then*

$$\bar{C}(x^*) = \frac{n-1}{2n-1}.$$

It is easy to verify that

$$\frac{1}{2} > \frac{n}{2n+1} > \frac{n-1}{2n-1}.$$

So, the additive rule is the most efficient among the three rules defined above.

Proof. (To be added) □

6. Cournot Duopoly

There are two firms, $i = 1, 2$ that produce homogeneous products. Each firm i can produce q_i units at the cost of $c_i q_i$. The marginal cost parameter c_i is firm i's private information. It is commonly known that $c_i \in [\underline{c}, \overline{c}]$, where $0 \le \underline{c} \le \overline{c} \le 1/2$.

The firms face the inverse demand curve given by $p = 1 - q_1 - q_2$. They choose q_1 and q_2 simultaneously to maximize their profits,

$$\pi_1(q_1, q_2) = (1 - q_1 - q_2 - c_1)q_1 \text{ and } \pi_2(q_1, q_2) = (1 - q_1 - q_2 - c_2)q_2.$$

6.1. **Bayesian Nash equilibrium.**

Claim 9. *There exists a unique Bayesian Nash equilibrium (q_1, q_2) given by*

$$q_i(c_i) = \frac{2 - 3c_i + C}{6}, \quad i = 1, 2,$$

where $C = \mathbb{E}[c_i]$.

Proof. Firm 1's expected profit is

$$\mathbb{E}[\pi_1(q_1, q_2)] = (1 - c_1 - q_1 - \mathbb{E}[q_2(c_2)])q_1$$

so

$$q_1(c_1) = \frac{1 - c_1 - \mathbb{E}[q_2(c_2)]}{2}.$$

By symmetry, $\mathbb{E}[q_2(c_1)] = \mathbb{E}[q_2(c_2)]$, so

$$\mathbb{E}[q_i(c_i)] = \frac{1 - \mathbb{E}[c_i] - \mathbb{E}[q_i(c_i)]}{2} = \frac{1 - C - \mathbb{E}[q_i(c_i)]}{2}$$

So

$$\mathbb{E}[q_1(c_1)] = \mathbb{E}[q_2(c_2)] = \frac{1 - C}{3}.$$

Cournot equilibrium:

$$q_i(c_i) = \frac{1 - c_i - (1 - C)/3}{2} = \frac{2 - 3c_i + C}{6}.$$

□

6.2. **Best compromise equilibrium.**

Claim 10. *There exists a unique best compromise equilibrium (q_1^*, q_2^*) given by*

$$q_i^*(c_i) = \frac{2 - 3c_i + (\underline{c} + \overline{c})/2}{6}, \quad i = 1, 2.$$

Proof. Let $q_i^*(c_i)$ be a strategy of player i.

Firm 1's maximal profit given the knowledge of q_2 is

$$\pi_1^*(c_1, q_2) = \max \pi_1(q_1, q_2) = (1 - c_1 - q_1 - q_2)q_1$$

so

$$q_1^* = \frac{1 - c_1 - q_2}{2},$$

so

$$\pi_1^*(c_1, q_2) = \frac{(1 - c_1 - q_2)^2}{4}.$$

The complaint is

$$\Delta\pi_1(c_1, q_1, q_2) = \pi_1^* - \pi_1 = \frac{(1 - c_1 - q_2)^2}{4} - (1 - c_1 - q_1 - q_2)q_1.$$

This is convex in q_2, so we consider the extreme ones, $\underline{q}_2 = \min_c q_2^*(c)$ and $\overline{q}_2 = \max_c q_2^*(c)$. It is easy to see that the maxmin complaint is attained when

$$\Delta\pi_1(c_1, q_1, \underline{q}_2) = \Delta\pi_1(c_1, q_1, \overline{q}_2),$$

so

$$q_1^*(c_1) = \frac{1}{2}\left(1 - c_1 - \frac{\underline{q}_2 + \overline{q}_2}{2}\right).$$

Using $\underline{q}_1 = q_1^*(\overline{c})$ and $\overline{q}_1 = q_1^*(\underline{c})$ and the symmetry, so $\underline{q}_1 = \underline{q}_2$ and $\overline{q}_1 = \overline{q}_2$, we solve

$$\underline{q} = \frac{1}{2}\left(1 - \overline{c} - \frac{\underline{q} + \overline{q}}{2}\right) \quad \text{and} \quad \overline{q} = \frac{1}{2}\left(1 - \underline{c} - \frac{\underline{q} + \overline{q}}{2}\right).$$

So

$$\underline{q} = \frac{1}{3} + \frac{\underline{c} - 5\overline{c}}{12} \quad \text{and} \quad \overline{q} = \frac{1}{3} + \frac{\overline{c} - 5\underline{c}}{12}.$$

(Note: $0 \le \underline{q} \le \overline{q} \le 1/2$ and so $p = 1 - q_1 - q_2 \ge 0$, because $0 \le \underline{c} \le c_i \le \overline{c} \le 1/2$.)

So, best compromise equilibrium:

$$q_i^*(c_i) = \frac{1}{3} + \frac{\underline{c} + \overline{c}}{12} - \frac{c_i}{2} = \frac{2 - 3c_i + \frac{\underline{c} + \overline{c}}{2}}{6}.$$

□

6.3. **Comparison.** Observe that the Bayesian Nash and best compromise equilibria coincide when c_i is uniform, so

$$\mathbb{E}[c_i] = C = \frac{\underline{c} + \overline{c}}{2}.$$

Let us compare best compromise and Bayesian Nash equilibria according to their ex-post efficiency losses.

Best compromise: Expected price under a given distribution of costs with mean C is

$$\mathbb{E}[p] = \mathbb{E}[1 - q_1^*(c_1) - q_2^*(c_2)] = \frac{1 + 3C - \frac{\underline{c} + \overline{c}}{2}}{3}.$$

Ex-post complaint of firm i is

$$\Delta\pi_i(c_i, c_j) = \frac{1}{16}\left(\frac{\underline{c}+\overline{c}}{2} - c_j\right)^2 \leq \frac{1}{64}(\overline{c}-\underline{c})^2.$$

The upper bound is attained by $c_j = \overline{c}$.

Bayesian Nash: Expected price under a given distribution of costs with mean C is

$$\mathbb{E}[p] = (1 - \mathbb{E}[q_1(c_1)] - \mathbb{E}[q_2(c_2)]) = \frac{1+2C}{3}.$$

Ex-post complaint of firm i is

$$\Delta\pi_i(c_i, c_j) = \frac{1}{16}(C - c_j)^2 \leq \frac{1}{16}(\overline{c}-\underline{c})^2.$$

The maximum complaint is attained by, e.g., $C = \overline{c}$ (so the firm believes that the opponent has the cost $\overline{c}$ almost surely), whereas the realized cost is $c_j = 0$.

The standard Bayesian Nash equilibrium has each firm's maximum complaint 4 times as big as that of the best compromise equilibrium.

Under a given distribution, the expected price is more sensitive to the mean $C = \mathbb{E}[c_i]$ under best compromise. So when $C < \frac{\underline{c}+\overline{c}}{2}$, the best compromise features a smaller price (i.e., more efficient as it is closer to marginal cost) than the standard Cournot. Conversely, when $C > \frac{\underline{c}+\overline{c}}{2}$, the best compromise features a greater price (i.e., less efficient) than the standard Cournot.

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